CIRCULAR FUNCTIONS AND TRIGONOMETRY

Question 1

Given that $0 \le \theta \le \frac{\pi}{2}$ and $\tan\theta = \frac{3}{4}$, find

(a) $\cos\theta$.

(b) $\sin 2\theta$.

(c) $\tan\left(\frac{\pi}{2} - \theta\right)$.

Question 2

Given that $0 < \theta \le \frac{\pi}{2}$, arrange, in increasing order, $\sin\theta$, $\frac{1}{\sin\theta}$, $\sin^2\theta$.

Question 3

(a) If $0 < \theta < 90°$ and $\cos\theta = \frac{1}{3}a$, find $\sin\theta$.

(b) Express $\frac{\pi}{5}$ in degrees.

(c) Evaluate $\cos 300° \cos 30°$

(d) Express in terms of $\tan\theta$, $\frac{\sin(\pi - \theta)}{\cos\left(\frac{\pi}{2} + \theta\right)} \cdot \tan\left(\frac{3\pi}{2} - \theta\right)$.

Question 4

(a) Express $\frac{1}{\cos\theta - 1} - \frac{1}{\cos\theta + 1}$ in terms of $\sin\theta$.

(b) Solve $\frac{1}{\cos\theta - 1} - \frac{1}{\cos\theta + 1} = -8, 0° < \theta < 360°$

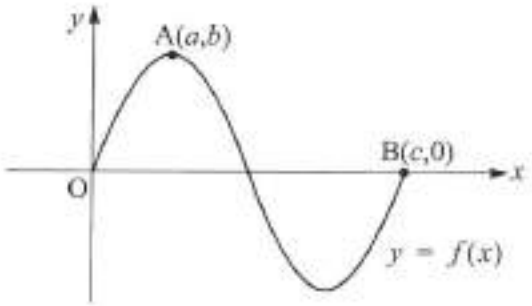
Question 5

(a) Given that $\cos\theta\sin\theta = \frac{1}{2}$, evaluate $(\cos\theta - \sin\theta)^2$.

(b) Find all values of θ such that (i) $\sin^2\theta - \sin\theta = 0, 0 \le \theta \le 2\pi$.
(ii) $\sin^2\theta - \sin\theta = 2, 0 \le \theta \le 2\pi$.

Question 6

The figure below shows the graph of $f(x) = 2\sin\left(\frac{x}{2}\right)$.

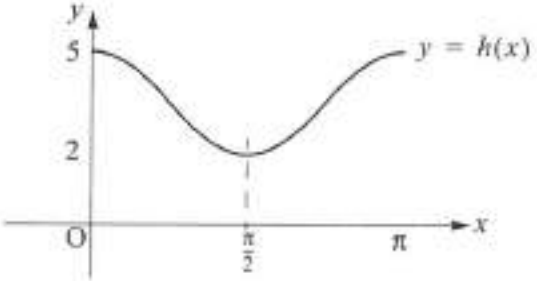


(a) Find a, b and c.

(b) Solve for x, where $f(x) = \sqrt{3}, 0 \le x \le c$.

Question 7

Consider the graph of the function $h(x) = a\cos(bx) + c$:



Find the values a, b and c.

Question 8

(a) State the range of $2\sin\theta$.

(b) Find (i) the smallest value of $\frac{1}{3+4^{\sin\theta}}$.

(ii) the largest value of $\frac{1}{3+4^{\sin\theta}}$.

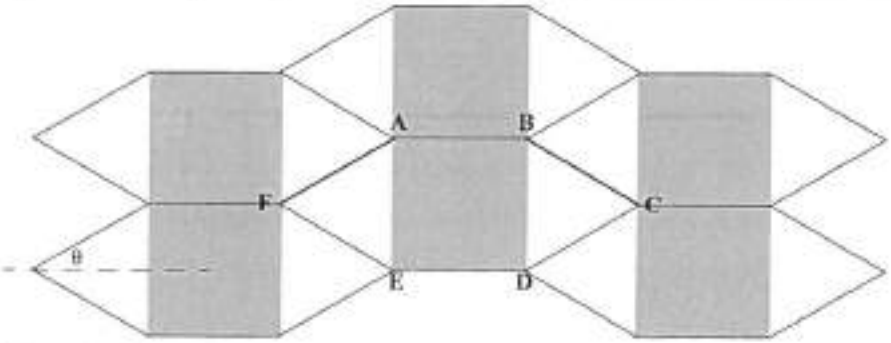
Question 9

(a) Find all values of x such that $2\cos(x)+\sin(2x) = 0, x \in [0, 2\pi]$.

(b) How many solutions to $2\cos(x)+\sin(2x) = \cos x$ are there in the interval $[0, 2\pi]$.

Question 10

Part of a particular tile pattern, made up by joining hexagonal shaped tiles, is shown below.



The side lengths of every hexagon is x cm.

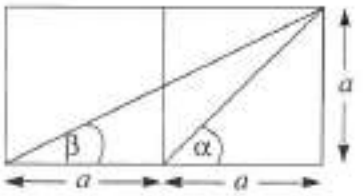
(a) Find in terms of x and θ the length of [AE].

(b) If the area of the shaded region in any one of the hexagons is 9 cm^2 and $\sin\theta = \frac{1}{8}$, find the value of x.

Question 11

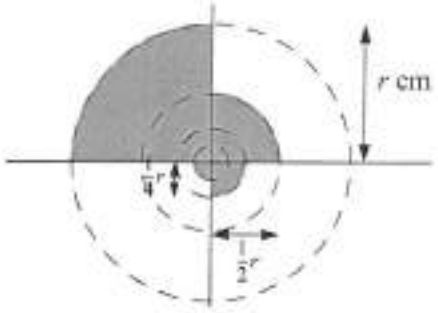
For the diagram shown alongside, the value of $\sin(\alpha-\beta) = \frac{1}{\sqrt{k}}$, where $k \in \mathbb{Z}^+$.

Find the value of k.



Question 12

The following diagram shows continually decreasing quarter circles, where each successive quarter circle has a radius half that of the previous one.



Let A_1 equal the area of the quarter circle of radius r, A_2 equal the area of the quarter circle of radius $\frac{1}{2}r$, A_3 equal the area of the quarter circle of radius $\frac{1}{4}r$ and so on.

(a) Find (i) A_1 in terms of r.

(ii) A_2 in terms of r.

(iii) A_3 in terms of r.

(b) If $A_1 + A_2 + A_3 = \frac{525}{64}\pi$, find r.

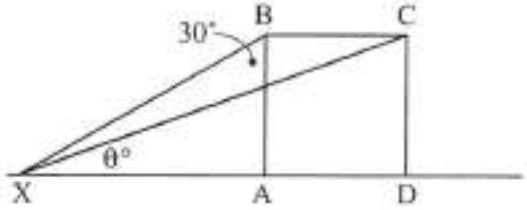
(c) If quarter circles are drawn and shaded in indefinitely, find r if $\sum_{i=1}^{\infty} A_i = 27\pi$.

Question 13

A pole of length b metres resting against a wall makes an angle of 60° with the ground. The end of the pole making contact with the ground starts to slip away from the wall until it comes to rest, 1 m from its initial position, where it now makes an angle of 30° with the ground. Find the value of b.

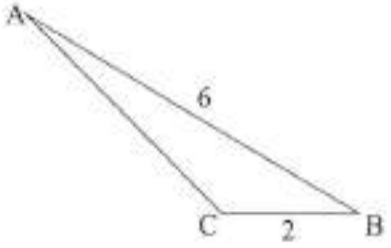
Question 14

The figure below shows a square ABCD. If $\angle XBA = 30°$ and $\angle CXA = \theta°$, find $\tan\theta°$.



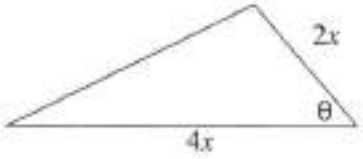
Question 15

In the diagram below, $\cos B = \frac{3}{8}$, AB = 6 and BC = 2, AC = $\sqrt{b}$, find b.



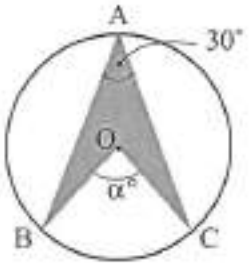
Question 16

The area of the triangle shown is 27 sq. units. Given that $\sin\theta = \frac{3}{4}$, find x.



Question 17

A circular badge of radius 2 cm has the following design:



(a) State the value of α.

(b) Find (i) the length of the minor BC.
(ii) the area of the minor sector OBC.

(c) Find the area of the shaded region shown.

Question 18

If $0° \le x \le 90°$, solve each of the following

(a) $\cos x = \sin 36°$.

(b) $\cos x = \sin x$.

(c) $\cos 2x = \sin x$.

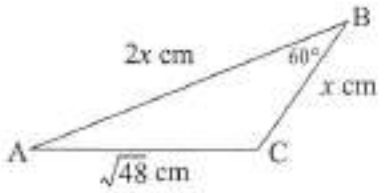
Question 19

(a) If $\tan\theta = a$, express $\dfrac{4\sin\theta}{5\cos\theta - \sin\theta}$ in terms of a.

(b) **Hence** find the value of θ if $\dfrac{4\sin\theta}{5\cos\theta - \sin\theta} = 1$, $0 < \theta < \dfrac{\pi}{2}$.

Question 20

Consider the triangle ABC:

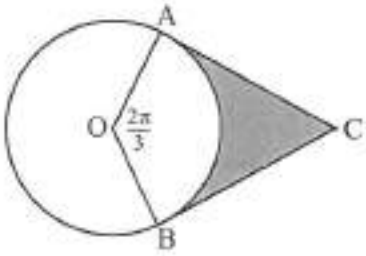


(a) Find x.

(b) Find the area of ΔABC

Question 21

The segments [CA] and [CB] are tangents to the circle at the points A and B respectively. If the circle has a radius of 4 cm and $\angle AOB = \dfrac{2\pi}{3}$.

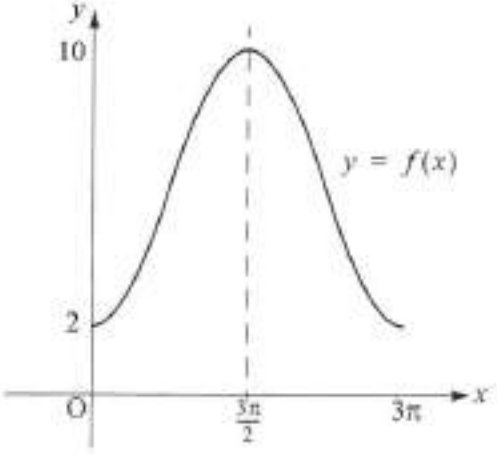


(a) Find the length of [OC].

(b) Find the area of the shaded region.

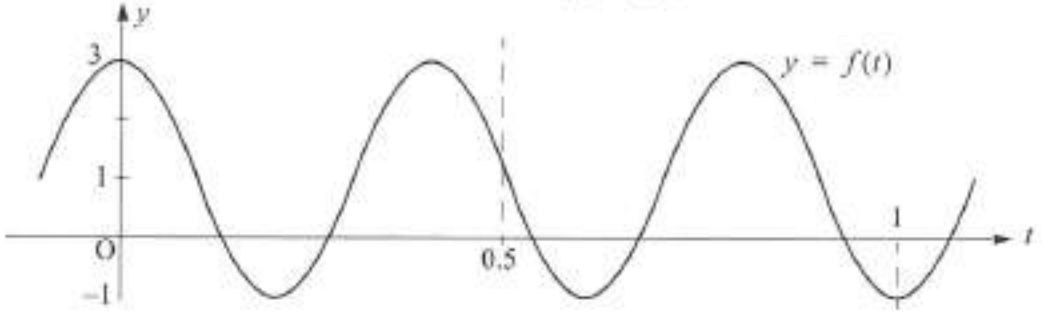
Question 22

The graph of $f(x) = a\cos(bx) + c$, $0 \le x \le \pi$ is shown below. Find the values a, b and c.



Question 23

(a) Part of the graph of the function defined by $y = f(t)$ is shown below.



(i) What is the period of $f(t)$?

(ii) Given that $f(t) = a\cos(kt) + c$, find a, k and c.

(b) What is the least positive value of x for which $\cos(2x) = \sqrt{3}\sin(2x)$?

(c) Let $g(x) = m\cos(x) + n$, $n, m \in \mathbb{R}$ and $m > 0$. Write an expression for n in terms of m if $g(x) < 0$ for all real values of x.

(d) Sketch the graph of $h :]-\pi,\pi[\mapsto \mathbb{R}$, where $h(x) = \tan\left(\frac{x}{2}\right) + 1$, clearly determining and labelling the x-intercepts.

(e) Find the sum of the solutions to $\cos\left(\frac{x}{2}\right) = \frac{1}{2}\sqrt{3}$, $x \in [0, 4\pi]$.

Question 24

Destructive interference generated by two out of tune violins results in the production of a sound intensity, $I(t)$, given by the equation

$$I(t) = 30 + 4\cos\left(\frac{4\pi}{3}t\right) \text{ dB (decibels)}$$

where t is the time in seconds after the violins begin to sound.

(a) What is the intensity after 1 second?

(b) What is the least intensity generated?

(c) When does the intensity first reach 32 dB?

(d) What time difference exists between successive measures of maximum intensity?

Question 25

The rabbit population, $N(t)$, over a ten year cycle in a small region of South Australia fluctuates according to the equation

$$N(t) = 950\cos(36t^\circ) + 3000,\ 0 \le t \le 10$$

where t is measured in years.

(a) Find the rabbit population after 2 and a half years.

(b) What is the minimum number of rabbits in this region that is predicted by this model?

(c) Sketch the graph of $y = N(t)$, $0 \le t \le 10$.

(d) For how long, over a 10 year cycle, will the rabbit population number at most 3475?

Question 26

Solve the equation for x, where $\sin 2x = \sqrt{3}\cos x,\ 0 \le x \le \pi$.

CIRCULAR FUNCTIONS AND TRIGONOMETRY

1. (a) $\frac{4}{5}$ (b) $\frac{24}{25}$ (c) $\frac{4}{3}$

2. $\sin^2\theta < \sin\theta < \frac{1}{\sin\theta}$

3. (a) $\frac{\sqrt{9-a^2}}{3}$ (b) $36°$ (c) $\frac{\sqrt{3}}{4}$ (d) $-\cot\theta$

4. (a) $-\frac{2}{\sin^2\theta}$ (b) $30°, 150°, 210°, 330°$

5. (a) 0 (b) (i) $0, \frac{\pi}{2}, \pi, 2\pi$ (ii) $\frac{3\pi}{2}$

6. (a) $a = \pi, b = 2, c = 4\pi$ (b) $\frac{2\pi}{3}, \frac{4\pi}{3}$

7. $a = \frac{3}{2}, b = 2, c = \frac{7}{2}$

8. (a) $[-2, 2]$ (b) (i) $\frac{1}{7}$ (ii) $\frac{4}{13}$

9. (a) $\frac{\pi}{2}, \frac{3\pi}{2}$ (b) 4

10. (a) $2x\sin\theta$ (b) 6

11. 10

12. (a) (i) $\frac{1}{4}\pi r^2$ (ii) $\frac{1}{16}\pi r^2$ (iii) $\frac{1}{64}\pi r^2$ (b) 5 (c) 9

13. $b = \sqrt{3} + 1$

14. $\frac{1}{2}(3 - \sqrt{3})$

15. 31

16. 3

17. (a) $60°$ (b) (i) $\frac{2\pi}{3}$ cm (ii) $\frac{2\pi}{3}$ cm^2 (c) 2 cm^2

18. (a) $54°$ (b) $45°$ (c) $30°$

19. (a) $\frac{4a}{5-a}$ (b) $\frac{\pi}{4}$

20. (a) 4 (b) $8\sqrt{3}$ cm^2

21. (a) 8 cm (b) $\frac{16}{3}(3\sqrt{3} - \pi)$ cm^2 22. $a = -4, b = \frac{2}{3}, c = 6$

23. (a) (i) 0.4 (ii) $a = 2, k = 5\pi, c = 1$ (b) $\frac{\pi}{12}$ (c) $n < -m$
(d) See soln (e) 4π

24. (a) 28 dB (b) 26 dB (c) 0.25 sec (d) 1.5 sec

25. (a) 3000 (b) 2050 (c) See soln (d) 20/3 yrs

26. $\frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}$