

FUNCTIONS AND EQUATIONS

Question 1

Consider the function $g(x) = \frac{1}{\sqrt{x-3}}$, $x \in X$.

- (a) Find (i) $g(4)$,
 (ii) $g(12)$,
 (iii) $g(9)$.
- (b) Find the largest set X for which $g(x)$ is defined.
- (c) Sketch the graph of $g(x)$ using its maximal domain X .

Question 2

Consider the functions $f(x) = \frac{1}{x-1}$ and $g(x) = x^3$.

- (a) Determine the maximal domain of (i) $f(x)$,
 (ii) $g(x)$.
- (b) (i) Justify the existence of $(g \circ f)(x)$.
 (ii) Find $(g \circ f)(x)$.

Question 3

Consider the function $f(x) = \sqrt{7-x}$.

- (a) Find (i) $f(3)$,
 (ii) $f(7)$.
- (b) What is the maximal domain of $f(x)$.
- (c) Find (i) $g(x) = \frac{1}{f(x)}$ and determine the implicit domain of $g(x)$,
 (ii) $h(x) = f^{-1}(x)$ and determine the implicit domain of $h(x)$.

Question 4

Given the function $f(x) = \frac{1}{1 + e^{2x}}$, $x \in \mathbb{R}$ and the fact that $f(x) + f(-x) = ax + b$, where $a, b \in \mathbb{R}$, find the values of a and b .

Question 5

Consider the function $f(x) = 3x^2 - 6x + 7$.

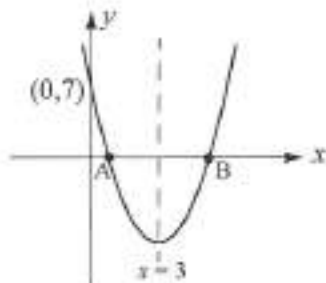
- Express $f(x)$ in the form $a(x - h)^2 + k$, where $a, b, c \in \mathbb{Z}^+$.
- For the graph of $f(x)$ write down the
 - coordinates of its vertex.
 - equation of the axis of symmetry.
- Find the coordinates of the y -intercept.
 - Sketch the graph of $f(x)$, clearly labelling the vertex, y -intercept and axis of symmetry.

Question 6

- Determine the maximal domain of $f(x) = \ln(x + 1)$.
- If $g(x) = e^x$,
 - justify the existence of $(f \circ g)(x)$.
 - find $(f \circ g)(x)$.
 - state the maximal domain of $(f \circ g)(x)$.

Question 7

The graph of the function $g(x) = 2x^2 - 12x + 7$ is shown below. The coordinates of B are $(a + \sqrt{\frac{b}{c}}, 0)$ where $a, b, c \in \mathbb{Z}$. Find the values of a, b and c .



Question 8

Consider the function $g(x) = x + \frac{1}{x}$, $x > 0$.

- (a) (i) Find $g(2)$.
 (ii) Find the x -value for which $g(x) = 2$.
- (b) The composite function $(g \circ g)(x) = \frac{f(x)}{x(1+x^2)}$, $x > 0$. Find $f(x)$.

Question 9

- (a) Find the $(f \circ g)(x)$ where $g(x) = e^x + 1$, $x \in \mathbb{R}$ and $f(x) = (x-1)\ln(x-1)$, $x > 1$.
- (b) Find $\{x : (f \circ g)(x) = 2x\}$.
- (c) What would the solution to $(f \circ g)(x) = 2x$ be if $g(x) = e^x + 1$, $x > 0$?

Question 10

- (a) Find the inverse function, f^{-1} , of $f(x) = \frac{1}{x-2}$, $x > 2$.
- (b) On the same set of axes, sketch the graphs of $f(x)$ and $f^{-1}(x)$.
- (c) Find the coordinates of the point of intersection of $f(x)$ and $f^{-1}(x)$.

Question 11

Consider the function $f_k(x) = \frac{e^{-kx}}{2 + e^{-x}}$, $x \in \mathbb{R}$.

- (a) Find (i) $\{x : f_2(x) = 1\}$.
 (ii) $\left\{x : f_1(x) = \frac{1}{2}\right\}$.
- (b) For what values of a will $\left\{x : f_1(x) = \frac{1}{a}\right\} = \mathbb{R}$.

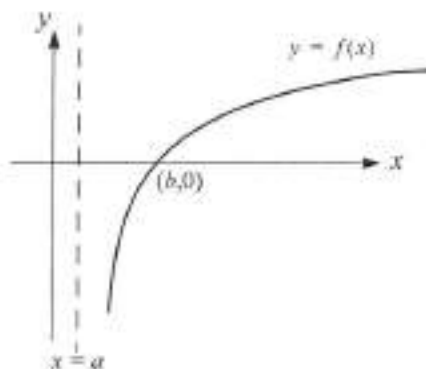
Question 12

- (a) On the same set of axes, sketch the graphs of $f(x) = x^2 - 4$ and $g(x) = \sqrt{x-2}$.
- (b) What is the domain of $(f \circ g)(x)$?
- (c) Find (i) $(f \circ g)(x)$,
(ii) the range of $(f \circ g)(x)$.

Question 13

The graph of $f(x) = -1 + \ln(x-1)$, $x > a$ is shown.

- (a) Find the values of a and b .
- (b) Find $f^{-1}(x)$.
- (c) Sketch the graph of $f^{-1}(x)$.



Question 14

Find the maximal domain of $h(x) = \frac{1}{\ln(\ln x)}$.

Question 15

- (a) On the same set of axes sketch the graphs of $f(x) = 5x + 2$ and $g(x) = 3x^2$.
- (b) Find (i) $\{x : f(x) = g(x)\}$,
(ii) $\{x : 3x^2 \leq 5x + 2\}$.

Question 16

Consider the function $f : A \rightarrow \mathbb{R}$, where $f(x) = \log_3(3x + 2)$.

- (a) Find the largest set A for which f is defined.
- (b) (i) Define fully, the inverse, f^{-1} ,
(ii) Sketch the graph of $f^{-1}(x)$,
(iii) If $g(x) = 3^x$, express $f^{-1}(x)$ in terms of $g(x)$.

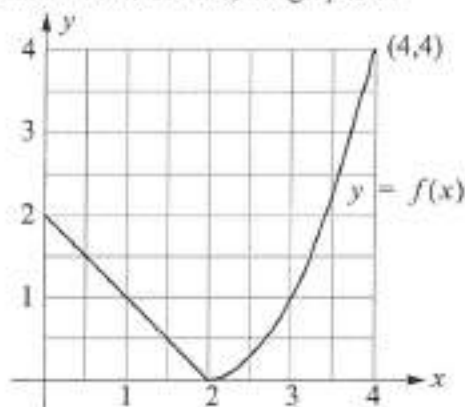
Question 17

For the graph shown below, sketch, on different sets of axes, the graphs of

(a) $y = f(x+1)$.

(b) $\frac{1}{2}y = f(x)$.

(c) $y = f(x) - 2$.



Question 18

Let $f(x) = \frac{1}{x}$, $x \neq 0$. The graph of $g(x)$ is a translation of the graph of $f(x)$ defined by the matrix $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

- (a) Find an expression for $g(x)$
- in terms of $f(x)$,
 - in terms of x .
- (b) On the same set of axes, sketch the graphs of
- $f(x)$,
 - $g(x)$, for $x > 3$.

Question 19

The function $g(x) = ax - b$ passes through the points with coordinates $(1, 9)$ and $(-3, 1)$. Find a and b .

Question 20

Let $f(x) = x^2, x \in \mathbb{R}$.

- (a) Sketch the graph of f .

The graph of f is transformed to the graph of g by a translation $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$.

- (b) Find an expression for $g(x)$ in terms of

- (i) $f(x)$,
 (ii) x .

- (c) (i) Find $\{x : g(x) = 0\}$.
 (ii) Find $g(0)$.
 (iii) Sketch the graph of $g(x)$.

Question 21

Given that $f(x) = \frac{x-1}{x+1}$, find (a) $f(1)$.

- (b) $f^{-1}(-1)$.

Question 22

The function $f(x)$ undergoes a transformation defined by the matrix $\begin{pmatrix} a \\ -b \end{pmatrix}$ to produce the new function $g(x)$.

- (a) Express $g(x)$ in terms of $f(x)$.
 (b) If $f(x) = x^2 - 2x + 4$ and $g(x) = x^2 - 2x + 8$, find a and b .

Question 23

Let $f(x) = 2e^{-x} + 3, x \in \mathbb{R}$, find (a) $f(-1)$.

- (b) $f^{-1}(4)$.

Question 24

Let $f(x) = x^3$ and $g(x) = x - 1$. Find

- (a) $(f \circ g)(1)$.
- (b) $(g \circ f)(2)$.

Question 25

Consider the function $f(x) = \begin{cases} x - 2, & \text{if } x < 0 \\ (x - 1)^2 - 3k, & \text{if } x \geq 0 \end{cases}$.

- (a) Find the value of k for which $f(x)$ is continuous for $x \in \mathbb{R}$.
- (b) Using the value of k in (a), sketch the graph of $f(x)$, clearly labelling all intercepts with the axes.

Question 26

Find all real values of k so that the graph of the function $h(x) = 2x^2 - kx + k$, $x \in \mathbb{R}$ cuts the x -axis at two distinct points.

FUNCTIONS AND EQUATIONS

1. (a) (i) 1 (ii) $\frac{1}{3}$ (iii) $\frac{1}{6}\sqrt{6}$ (b) $x > 3$ (c) See soln.
2. (a) (i) $\mathbb{R} \setminus \{1\}$ (ii) \mathbb{R} (b) (ii) $(g \circ f)(x) = \frac{1}{(x-1)^3}, x \neq 1$
3. (a) (i) 2 (ii) 0 (b) $x \leq 7$ (c) (i) $]-\infty, 7[$ (ii) $[0, \infty[$
4. $a = 0, b = 1$
5. (a) $f(x) = 3(x-1)^2 + 4$ (b) (i) (1, 4) (ii) 1
(c) (i) (0, 7) (ii) See soln.
6. (a) $]-1, \infty[$ (b) (ii) $(f \circ g)(x) = \ln(e^x + 1)$ (iii) \mathbb{R}
7. $a = 3, b = 11, c = 2$
8. (a) (i) 2.5 (ii) 1 (b) $f(x) = x^4 + 3x^2 + 1$
9. (a) $(f \circ g)(x) = xe^x, x \in \mathbb{R}$ (b) 0, $\ln 2$ (c) $\ln 2$
10. (a) $f^{-1}(x) = \frac{1}{x} + 2, x \in]0, \infty[$ (b) See soln. (c) $(1 + \sqrt{2}, 1 + \sqrt{2})$
11. (a) (i) $-\ln 2$ (ii) $-\ln 2$ (b) $a > 1$
12. (a) See soln. (b) $[2, \infty[$ (c) (i) $(f \circ g)(x) = x - 6, x \geq 2$ (ii) $[-4, \infty[$
13. (a) $a = 1, b = 1 + e$ (b) $f^{-1}(x) = 1 + e^{x+1}, x \in \mathbb{R}$ (c) See soln.
14. $x > 1, x \neq e$
15. (a) See soln. (b) (i) $\left\{2, -\frac{1}{3}\right\}$ (ii) $\left\{x \mid -\frac{1}{3} \leq x \leq 2\right\}$
16. (a) $\left\{x \mid x > \frac{2}{3}\right\}$ (b) (i) $f^{-1}(x) = 3^{x-1} - \frac{2}{3}, x \in \mathbb{R}$ (ii) See soln.
(iii) $f^{-1}(x) = \frac{1}{3}(g(x) - 2)$
17. See soln.
18. (a) (i) $g(x) = f(x-3) + 2$ (ii) $g(x) = \frac{1}{x-3} + 2, x \neq 3$ (b) See soln
19. $a = 2, b = -7$
20. (a) See soln (b) (i) $g(x) = f(x+1) + 2$ (ii) $g(x) = (x+1)^2 + 2$
(c) (i) \emptyset (ii) 3 (iii) See soln
21. (a) 0 (b) 0
22. (a) $g(x) = f(x-a) - b$ (b) $a = 0, b = -4$
23. (a) $2e + 3$ (b) $\ln 2$
24. (a) 0 (b) 7
25. (a) 1 (b) See soln
26. $]-\infty, 0[\cup]8, \infty[$