

ALGEBRA

Question 1

An arithmetic sequence is defined in such a way that the 15th term, $u_{15} = 92$, and the 3rd term, $u_3 = 56$.

- (a) Determine
 (i) the common difference.
 (ii) the first term.
 (b) Calculate S_{12} .

Question 2

A geometric sequence is such that its 4th term, $u_4 = 135$ and the ratio $\frac{u_9}{u_4} = 3^5$.

- (a) Find (i) the common ratio.
 (ii) the first term.
 (b) If $S_{10} = a(b - 1)$, determine the real constants a and b .

Question 3

Consider the sequence defined by the equation $u_n = 1 - 9n$, $n = 1, 2, 3, \dots$, where u_n represents the n th term of the sequence.

- (a) Write down the value of u_1 , u_2 and u_3 .
 (b) Calculate $\sum_{n=1}^{10} (1 - 9n)$.

Question 4

Solve for x in each of the following equations

- (a) $\log_2 x^4 = \log_2 16$.
 (b) $\log_x 27 = 3$.
 (c) $\log_4 32 = x$.
 (d) $\log_3(1 - x) - \log_3 x = 2$.

Question 5

The n th term of a sequence is given by

$$u_n = \frac{2}{3} \times 3^n, n = 1, 2, 3, \dots$$

If $\sum_{n=1}^{20} \left(\frac{2}{3} \times 3^n\right) = a(3^{20} - b)$, determine the values of a and b .

Question 6

Solve the following equations for x .

- (a) $x - 4 = 4$.
- (b) $|x - 4| = 4$.
- (c) $\log_2(x - 4) = 4$.
- (d) $x^2 - 4 = 4$.
- (e) $\log_2(x - 2) + \log_2(x + 2) = 2$.

Question 7

Solve the following equations for x .

- (a) (i) $2x - 1 = 14$.
- (ii) $\frac{3 - x}{2} = x$.
- (b) $(x - 2)(x + 1) = 1$.

Question 8

- (a) (i) Factorise the quadratic expression $x^2 - 6x + 5$.
- (ii) Solve the inequality for x , where $x^2 - 6x + 5 > 0$.
- (b) For what values of x is
 - (i) $\log_{10}(6 - x)$ defined?
 - (ii) $\log_{10}(x - 4)$ defined?
- (c) Solve for x , the equation $\log_{10}(x^2 - 6x + 5) = \log_{10}(6 - x) + \log_{10}(x - 4)$.

Question 9

- (a) (i) Expand $(a + 1)(a^2 + 2a - 1)$.
(ii) Solve for a the equation $a^3 + 3a^2 + a - 1 = 0$.
- (b) Solve for x where $(\ln x)^3 + 3(\ln x)^2 + \ln x = 1$.

Question 10

- (a) (i) Expand $(x + a)(x + 1)$.
(ii) Factorise $x^2 + 3x + 2$.
- (b) Solve each of the following equations for x ,
(i) $x^2 - (e + 1)x + e = 0$.
(ii) $e^{2x} - (e + 1)e^x + e = 0$.

Question 11

The first three consecutive terms of an arithmetic sequence are $k - 2$, $2k + 1$ and $4k + 2$.

- (a) Find the value of k .
- (b) Find (i) u_{10} .
(ii) S_{10} .

Question 12

The first three terms of a geometric sequence are given by $\sqrt{3} + 1$, x and $\sqrt{3} - 1$, where $x > 0$.

- (a) Find x .
- (b) Find S_{∞} .

Question 13

Given that $\frac{1}{2} + 1 + 2 + 2^2 + \dots + 2^{10} = a \times 2^b + c$, find the values a , b and c .

Question 14

A geometric sequence is such that each term is equal to the sum of the two terms directly following it. Find the positive common ratio.

Question 15

An arithmetic sequence is such that $u_p = q$ and $u_q = p$.

- (a) Find, in terms of p and q , u_1 .
- (b) Find S_{p+q} .

Question 16

Solve each of the following equations for x .

- (a) $x^2 + x - 2 = 0$.
- (b) $(\ln(x))^2 - \ln\left(\frac{1}{x}\right) - 2 = 0$.
- (c) $2^x - 2 \times 2^{-x} + 1 = 0$.

Question 17

- (a) Expand $(2a + 1)(3 - a)$.
- (b) Solve
 - (i) $\log_{10}(2x - 5) + \log_{10}(x) = \log_{10}3$.
 - (ii) $2 \times 3^{2x-1} - 5 \times 3^{x-1} - 1 = 0$.

Question 18

- Find k if
- (i) $\log_2(2k - a) = 3$.
 - (ii) $\log_{10}k = 1 + \log_{10}5$.
 - (iii) $3 \ln 2 - 2 \ln 3 = -\ln k$.

Question 19

- (a) If $\log(a - b) = \log a - \log b$, find an expression for a in terms of b , stating any restrictions on b .
- (b) If $a^2 + b^2 - 2ab = 4$ and $ab = 4$, where $a > 0$, $b > 0$, find the value of $2 \log_{20}(a + b)$.

ALGEBRA

1. (a) (i) 3 (ii) 50 (b) 798
2. (a) (i) 3 (ii) 5 (b) $a = \frac{5}{2}, b = 3^{10}$
3. (a) $u_1 = -8, u_2 = -17, u_3 = -26$ (b) -485
4. (a) ± 2 (b) 3 (c) 2.5 (d) 0.1
5. $a = 1, b = 1$
6. (a) 8 (b) 0, 8 (c) 20 (d) $\pm 2\sqrt{2}$ (e) $2\sqrt{2}$
7. (a) (i) 7.5 (ii) 1 (b) $\frac{1 \pm \sqrt{13}}{2}$
8. (a) (i) $(x-5)(x-1)$ (ii) $x < 1$ or $x > 5$
 (b) (i) $x < 6$ (ii) $x > 4$
 (c) $4 + \frac{1}{2}\sqrt{6}$
9. (a) (i) -1 or $-1 \pm \sqrt{2}$ (b) $e^{-1}, e^{-1 \pm \sqrt{2}}$
10. (a) (i) $x^2 + (1+a)x + a$ (ii) $(x+2)(x+1)$
 (b) (i) 1, e (ii) 0, 1
11. (a) 2 (b) (i) 45 (ii) 225
12. (a) $\sqrt{2}$ (b) $\frac{1}{2}(\sqrt{6} + 4 + \sqrt{2} + 2\sqrt{3})$
13. $a = 1, b = 11$ and $c = -\frac{1}{2}$ [or $a = \frac{1}{2}, b = 12, c = -\frac{1}{2}$ if $S_{12} = \frac{1}{2} \times 2^{12} - \frac{1}{2}$]
14. $r = \frac{-1 + \sqrt{5}}{2}$
15. (a) $u_1 = p + q - 1$ (b) $S_{p+q} = \frac{1}{2}(p+q)(p+q-1)$
16. (a) -2, 1 (b) e, e^{-2} (c) 0
17. (a) $-2a^2 + 5a + 3$ (b) (i) 3 (ii) 1
18. (i) $\frac{1}{2}(8+a)$ (ii) 50 (iii) $\frac{9}{8}$
19. (a) $a = \frac{b^2}{b-1}, b > 1$ (b) 1